

# ADM reduction of IIB on $\mathcal{H}^{p,q}$ and dS braneworld

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## Abstract

We propose a new Kaluza-Klein reduction scheme based on ADM decomposition. The scheme has been motivated by AdS/CFT, especially by how the worldvolume theory should appear from the supergravity side. We apply the scheme to IIB supergravity reduced on a 5D hyperboloidal  $\mathcal{H}^5$  space, and show that an (A)dS "braneworld" is realized after further reduction to 4D. We comment on applications to cosmology and black hole physics. In particular, the scheme should provide a proper paradigm for black hole physics.

# 1 Introduction

AdS/CFT belongs to a class of dualities in which the dualization procedure is not explicitly introduced. The reason for that is simple: implementing the procedure is subtle and difficult. Evidently, however, understanding the procedure must be essential for the first principle derivation of AdS/CFT. There has been progress in this direction (see, e.g., [1] [2] and refs therein), and one of the goals in this work is to further that progress. While doing so, we report on two conceptual/technical advances: the first is a new Kaluza-Klein reduction procedure based on ADM decomposition and the other is the IIB realization of a braneworld.

One of the distinctions of AdS/CFT type dualities is that the dualization and inverse dualization seem very different at low energy field theory level. We refer to the procedure in which one gets the closed string/gravity degrees of freedom from open string/gauge theory as *forward dualization*. The *inverse dualization* refers to the reverse procedure. It was proposed in [3] that it should be the quantum/strong coupling effects that must be behind the forward dualization. As commented in [4] (footnote 15), the inverse dualization must be initiated by a spontaneous symmetry breaking of the supergravity system. (Here we are using the term 'spontaneous symmetry breaking' in a general sense that is associated with expanding an action around a solution.<sup>1</sup>)

In the context of  $(A)dS/CFT$  type dualities, it is natural to view the  $(A)dS_5$  as a foliation of  $(A)dS_4$  along a direction of  $(A)dS_5$ . (See the figure.) One of the leaves can serve as our braneworld with a certain gravity localization mechanism that we discuss later. We focus on the  $dS$  case henceforth whenever possible. Figure 1 depicts  $dS_5$  as foliation of  $dS_4$  hypersurfaces. One may describe the  $dS_5$  through bulk gravity setup. On the other hand, it seems plausible to describe the bulk  $dS_5$ , or at least some aspects of it, through collective dynamics of the hypersurfaces. Of course, the existence of these "dual" descriptions must be what is behind gauge/gravity correspondence. (The bulk dynamics would include the dynamics associated with the "radial" direction which is typically associated with renormalization group flow. Therefore, in general, the collective surface dynamics would not cover the entire bulk dynamics. There are various levels of equivalence that the

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<sup>1</sup>The symmetries that are broken and the breaking patterns will not be pursued in this work.

term "duality" describes. Ideally, the term should be reserved only for the cases where the two theories under consideration are fully equivalent. For example, a canonical transformation can be viewed as a precise duality: it maps to a theory that is fully dual to the original theory. However, even if the full equivalence is not obvious, the term "duality" is often used in some string theory contexts for the cases in which the two theories capture substantial aspects each other.)

What procedure could lead to the surface degrees of freedom starting from the bulk theory? As anticipated in [4], it should be a procedure initiated by a spontaneous symmetry breaking. It is also likely that the procedure should involve a certain dimensional reduction scheme, conventional or unconventional. Although it should be possible to deduce the hypersurface degrees of freedom through the conventional Kaluza-Klein reduction (see, e.g., [5] for a relatively recent discussion), we will pave our way through an unconventional reduction scheme. This procedure of acquiring surface degrees of freedom should be viewed as a novel Kaluza-Klein (KK) reduction - what we call ADM reduction. What is unusual about this scheme is that the reduced lagrangian is not a gravity system: the *dynamical* fields are the worldvolume (i.e., the selected hypersurface) gauge fields. This phenomenon, unusual in the Kaluza-Klein context, must be what triggers the inverse dualization of the AdS/CFT type dualities.

An essential computational ingredient for obtaining the hypersurface degrees of freedom from a spontaneous symmetry breaking was obtained in a remarkable series of papers, [6–8]. The authors showed that the Hamilton-Jacobi equation of the gravity system [9] [10] [11] under consideration admits a solution of the worldvolume theory form. We will apply the technique of [6–8, 12] to a specific setup of the 5D gravity that can be obtained by reducing IIB supergravity on a 5D hyperboloidal space  $\mathcal{H}^{p,q}$ ,  $p + q = 5$  considered in [13] (and also on  $S^5$ ).

Once the 5D (A)dS gravity is obtained by reducing IIB supergravity on a 5D hyperboloidal space  $\mathcal{H}^5$  ( $S_5$ ) [13], a canonical transformation can be performed on the system to convert it into an equivalent, therefore dual, formulation that still takes the form of a supergravity. Following [6–8], one can show that the dual system admits, in the case of  $S^5$  reduction, a worldvolume action as solution of the Hamilton-Jacobi equation of the gravity system. Obviously, the resulting worldvolume action will capture some aspects of the original gravity theory, and can be viewed as a dual pair at least in a wider

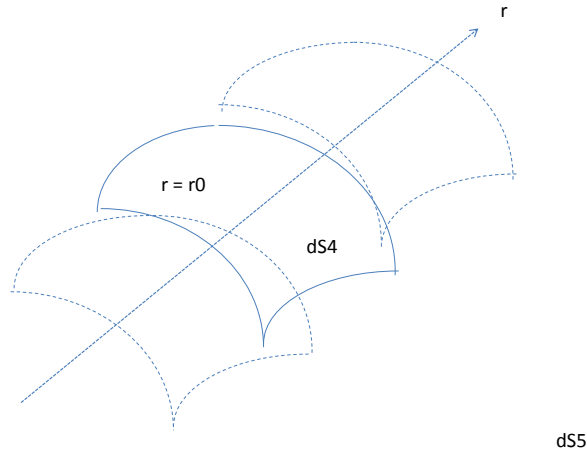


Figure 1:  $dS_5$  as foliation of  $dS_4$

sense of the term mentioned previously. In essence, the holographic dualities must have their roots in that one may adopt two different approaches to describe the geometry. In the first approach, one can adopt the conventional degrees of freedom, the metric, to describe the bulk physics. In the other approach, one slices the bulk into a set of hypersurfaces, i.e., focus on each leaf of the foliated geometry. Gravity is not needed as *dynamical* degrees of freedom to describe the hypersurface, although it serves as a background for the gauge degrees of freedom. We will elaborate on this in the main body of this paper.

Below we consider both the ADM reduction and the standard toroidal-type reduction. In the ADM reduction scheme, one employs the HJ procedure, and a worldvolume effective action will appear as a solution of the HJ equation. The ADM reduction scheme can be viewed as "emergent gauge theory" in the sense that a worldvolume gauge field emerges from the symmetry breaking. In the standard toroidal reduction, dependence of one of the coordinates ("r") will be removed. The model that we obtain below has a scalar that can be viewed as an inflaton field in four dimensions. We comment on the potential phenomenological value of our model in the main

body postponing the full analysis for the near future. In the related literature, usually, an explicit coupling between gravity and various brane sources is employed followed by Calabi-Yau compactification in order to obtain a de Sitter space in the lower dimensions. One drawback of the Calabi-Yau compactification is the implicit nature of the analysis involved. Moreover, the original motivation for considering the Calabi-Yau manifold as opposed to a maximally symmetric manifold has diminished with better understanding of supersymmetry breaking effects of D-branes.

The organization of the paper is as follows: in sec 2, we carry out reduction of IIB supergravity on a manifold denoted by  $\mathcal{M}^5$  that we take either  $\mathcal{M}^5 = \mathcal{H}^5$  or  $\mathcal{M}^5 = S^5$  (5-sphere) in the subsequent sections. For  $\mathcal{M} = \mathcal{H}^5$ , we obtain a 5D de Sitter gravity. The ADM reduction of the 5D theory obtained thereby is carried out in sec 3. In particular, we elaborate on the appearance of the worldvolume gauge field. We discuss the implications of our results on braneworld realization and black hole information physics. In sec 4, we obtain a domain-wall solution for the 5D system obtained in sec 2 in the case  $\mathcal{M}^5 = S^5$ . Keeping the minimal set of fields, the system admits an AdS vacuum solution. We comment on possibility of obtaining a dS solution with the form fields added. In another direction, we carry out toroidal reduction to 4D, and obtain an action that may have phenomenological value for inflationary physics. In the conclusion, the results are summarized and future directions are suggested. We also comment on the potential cosmological/black hole applications of our results.

## 2 Spherical/hyperboloidal reduction in Einstein frame to 5D

Although mathematically elegant, the usual Calabi-Yau compactification has one shortcoming: the requirement of the internal manifolds to be of CY is not sufficiently restrictive, an aspect that can be seen from the fact that there exists many moduli. Starting with simple compactification such as compactification on a maximally symmetric space could be more effective. In the KKLT [14] type compactification, one introduces and/or adds (anti)-branes to lift up the moduli. This can be viewed as a narrowing-down to special sectors of the moduli space. Therefore, this approach ultimately

might not be more general than the present approach where one restricts to a certain class of special internal manifolds from the beginning.

In this section, we consider Kaluza-Klein reduction on an inhomogeneous<sup>2</sup> hyperboloidal space  $\mathcal{H}^{p,q}$ , a manifold considered in [13]. The authors of [13] showed that reduction on  $\mathcal{H}^{p,q}$  leads to a ghost-free dS gravity in the lower dimensions. The ansatz of [13] led to a 5D potential that has a saddle shape. In the reduction that we carry out in this section, we consider an ansatz that is an analogue of those of [6] keeping three scalars for 5D system: the dilaton, the axion and a breathing mode from the 10D metric. Only the breathing mode generates the potential for 5D theory as we will see below.<sup>3</sup>

The bosonic part of type IIB supergravity action takes the following form in Einstein frame [15]

$$\begin{aligned}
I = \frac{1}{2\kappa_{10}^2} \int d^{10}X \sqrt{-G_E} & \left[ \left( R - \frac{1}{2} \partial_M \Phi \partial^M \Phi - \frac{1}{2 \cdot 3!} e^{-\Phi} (H_{(3)})^2 \right) \right. \\
& - \frac{1}{2} e^{2\Phi} \partial_M C_{(0)} \partial^M C_{(0)} - \frac{1}{2 \cdot 3!} e^{\Phi} (\tilde{F}_{(3)})^2 - \frac{1}{4 \cdot 5!} (\tilde{G}_{(5)})^2 + \mathcal{L}_{PST} \Big] \\
& - \frac{1}{4\kappa_{10}^2} \int_{\mathcal{M}^{10}} C_{(4)} \wedge H_{(3)} \wedge F_{(3)}
\end{aligned} \tag{2.1}$$

Let us consider the following reduction ansatz<sup>4</sup>,

$$\begin{aligned}
ds_{10}^2 &= e^{2\tilde{\rho}(\hat{x})} h_{\underline{mn}}(\hat{x}) d\hat{x}^{\underline{m}} d\hat{x}^{\underline{n}} + e^{-6\tilde{\rho}(\hat{x})/5} d\Omega_5 \equiv e^{2\tilde{\rho}(\hat{x})} h_{\underline{mn}}(\hat{x}) d\hat{x}^{\underline{m}} d\hat{x}^{\underline{n}} + e^{-6\tilde{\rho}(\hat{x})/5} g_{ij} dy^i dy^j \\
\Phi(X) &= \phi(\hat{x})
\end{aligned}$$

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<sup>2</sup>Here inhomogeneity refers to the fact that the space is not a coset manifold.

<sup>3</sup>The authors of [6] considered IIB action in string frame. The Einstein frame ansatz that we consider below are not connected to the ansatz considered in [6], therefore should belong to a different class of ansatz. ( $S_5$  vs  $\mathcal{H}_5$  does not matter for this matter.) As a matter of fact, one can show that the precise forms of (2.4), (2.5) and (2.6) of [6] used in *Einstein* frame also lead to consistent reduction. (The precise forms of (2.4-6) of [6] and our ansatz (2.2), (2.3) lead to the same 5D action up to a numerical rescaling, namely, (2.11).) This seems rather unusual, and must be attributed to the simplicity of the ansatz. (Our earlier false accusations of the work of [6] were made in part by this subtlety.) In general, if an ansatz leads to consistent reduction in one frame, it would not in the other frame. In one of the footnotes in section 3, we point out another related aspect of the two ansatz.

<sup>4</sup>In a typical Kaluza-Klein reduction, it is usually a scalar sector that makes the procedure complicated. An example of sphere reduction with many scalars can be found in [17].

$$\begin{aligned}
B_{(2)}(X) &= \frac{1}{2} B_{\underline{mn}}(\hat{x}) d\hat{x}^{\underline{m}} \wedge d\hat{x}^{\underline{n}} \equiv B_{(2)}(\hat{x}) \\
C_{(0)}(X) &= \chi(\hat{x}) \\
C_{(2)}(X) &= \frac{1}{2} C_{\underline{mn}}(\hat{x}) d\hat{x}^{\underline{m}} \wedge d\hat{x}^{\underline{n}} \equiv C_{(2)}(\hat{x})
\end{aligned} \tag{2.2}$$

and

$$\begin{aligned}
C_{(4)}(X) &= \frac{1}{4!} D_{\underline{mnkl}}(\hat{x}) d\hat{x}^{\underline{m}} \wedge d\hat{x}^{\underline{n}} \wedge d\hat{x}^{\underline{k}} \wedge d\hat{x}^{\underline{l}} + \frac{1}{4!} k E_{i_1 i_2 i_3 i_4} dy^{i_1} \wedge dy^{i_2} \wedge dy^{i_3} \wedge dy^{i_4} \\
&\equiv D_{(4)}(\hat{x}) + k E_{(4)}(y)
\end{aligned} \tag{2.3}$$

The  $y$ -coordinates describe either  $\mathcal{H}^5$  or  $S^5$ :

$$\mathcal{M}^5 = \mathcal{H}^5 \quad \text{or} \quad S^5 \tag{2.4}$$

$\mathcal{M}^5$  is later decided to be  $S^5$  for the discussion in sec 3. The field  $E$  satisfies  $5\partial_{[i_1} E_{i_2 i_3 i_4 i_5]} = (1/\sqrt{g}) \epsilon_{i_1 i_2 i_3 i_4 i_5}$ . (Our form conventions are summarized in Appendix A.)

Substituting (2.2), (2.3) into 10D equation of motion, one can show after some algebra that the reduced field equations follow from the following 5D action:

$$\begin{aligned}
I = \frac{1}{2\kappa_5^2} \int d^5 \hat{x} \sqrt{-h} \Big[ &\left( R^{(5)} - \frac{24}{5} \partial_{\underline{m}} \tilde{\rho} \partial^{\underline{m}} \tilde{\rho} - \frac{1}{2} \partial_{\underline{m}} \phi \partial^{\underline{m}} \phi - \frac{1}{2 \cdot 3!} e^{-\phi-4\tilde{\rho}} (H_{(3)})^2 \right) \\
&- \frac{1}{2} e^{2\phi} \partial_{\underline{m}} \chi \partial^{\underline{m}} \chi - \frac{1}{2 \cdot 3!} e^{\phi-4\tilde{\rho}} (\tilde{F}_{(3)})^2 - \frac{1}{2 \cdot 5!} e^{-8\tilde{\rho}} (\tilde{G}_{(5)})^2 + e^{16\tilde{\rho}/5} R^{\mathcal{M}_5} \Big] \tag{2.5}
\end{aligned}$$

Above

$$\tilde{F}_{(3)} = F_{(3)} - \chi H_{(3)} \equiv dC_{(2)}(\hat{x}) - \chi(\hat{x}) \wedge dB_{(2)}(\hat{x}). \tag{2.6}$$

$$\begin{aligned}
\tilde{G}_{(5)} &= G_{(5)} - C_{(2)} \wedge H_{(3)}, \quad H = \frac{1}{2} \partial_{[\underline{m}} B_{\underline{n}\underline{k}]} d\hat{x}^{\underline{m}} \wedge d\hat{x}^{\underline{n}} \wedge d\hat{x}^{\underline{k}} \\
G_{(5)} &= \frac{1}{4!} \partial_{[\underline{m}_1} D_{\underline{m}_2 \dots \underline{m}_5]} d\hat{x}^{\underline{m}_1} \wedge \dots \wedge d\hat{x}^{\underline{m}_5}.
\end{aligned} \tag{2.7}$$

With rescaling

$$\tilde{\rho} = -\frac{1}{4} \sqrt{\frac{5}{3}} \rho \tag{2.8}$$

the action (2.5) becomes the following form with canonical kinetic terms:

$$I = \frac{1}{2\kappa_5^2} \int d^5 \hat{x} \sqrt{-h} \Big[ \left( R^{(5)} - \frac{1}{2} \partial_{\underline{m}} \rho \partial^{\underline{m}} \rho - \frac{1}{2} \partial_{\underline{m}} \phi \partial^{\underline{m}} \phi - \frac{1}{2 \cdot 3!} e^{-\phi + \sqrt{\frac{5}{3}} \rho} (H_{(3)})^2 \right) \right]$$

$$-\frac{1}{2}e^{2\phi}\partial_{\underline{m}}\chi\partial^{\underline{m}}\chi - \frac{1}{2\cdot 3!}e^{\phi+\sqrt{\frac{5}{3}}\rho}(\tilde{F}_{(3)})^2 - \frac{1}{2\cdot 5!}e^{2\sqrt{\frac{5}{3}}\rho}(\tilde{G}_{(5)})^2 + e^{-\sqrt{\frac{16}{15}}\rho}R^{\mathcal{M}_5} \Big] \quad (2.9)$$

Rescaling the  $\rho$  field further in (2.9)

$$\rho \rightarrow \sqrt{\frac{5}{3}}\rho \quad (2.10)$$

we arrive at an alternative form of the action:

$$I_5 = \frac{1}{2\kappa_5^2} \int d^5\xi \sqrt{-h} \left[ R^{(5)} - \frac{5}{6}(\partial\rho)^2 - \frac{1}{2}(\partial\phi)^2 - \frac{1}{2\cdot 3!}e^{-\phi}e^{\frac{5}{3}\rho}H_{(3)}^2 \right. \\ \left. - \frac{1}{2}e^{2\phi}(\partial\chi)^2 - \frac{1}{2\cdot 3!}e^{\phi}e^{\frac{5}{3}\rho}\tilde{F}_{(3)}^2 - \frac{1}{2\cdot 5!}e^{\frac{10}{3}\rho}\tilde{G}_{(5)}^2 + e^{-\frac{4}{3}\rho}R^{\mathcal{M}_5} \right] \quad (2.11)$$

### 3 ADM Reduction from 5D to 4D

In section 4, we will obtain a solution of a certain three brane configuration. As a matter of fact, the 5D system admits a whole class of D3-brane solutions as we will see. The first step is to obtain the Hamilton-Jacobi (HJ) equation pertaining to (3.5) through a series of manipulations following the works of [6–8]. The fact that a class of solutions of the HJ system of (3.5) takes a form of a DBI action has deep physical implications. For example, the steps for obtaining the worldvolume form solution to the HJ equation should be viewed as a realization of a reduction scheme. The reason is that the field equations that follow from the worldvolume action can be viewed as outcome of substituting an appropriately constructed Kaluza-Klein gravity ansatz into the 5D Hamilton-Jacobi equation. Therefore the whole procedure is in the usual spirit of Kaluza-Klein reduction; hence it can legitimately be called an ADM "reduction" scheme. Also the resulting D3 action should be a dual description, at least in the wider sense of the term "dual".

#### 3.1 Converting to 5D "string" type frame

In the next section, we consider the HJ equation of the 5D gravity system obtained in the previous section. It turns out more convenient for the purpose at hand to cast (2.11) into another frame which we call 5D "string" frame. To that end, let us consider

$$(h_{Ein})_{\underline{mn}} = e^{-\frac{1}{2}\phi+\frac{5}{6}\rho}(h_{str})_{\underline{mn}}; \quad (3.1)$$



With this, (2.11) now takes<sup>5</sup>

$$\begin{aligned}
I = & \frac{1}{2\kappa_5^2} \int d^5\hat{x} \sqrt{-h} \left[ e^{-\frac{3}{4}\phi + \frac{5}{4}\rho} \left( R^{(5)} - \frac{10}{3} \nabla^2 \rho - \frac{35}{12} \partial_{\underline{m}} \rho \partial^{\underline{m}} \rho + 2 \nabla^2 \phi \right. \right. \\
& - \frac{5}{4} \partial_{\underline{m}} \phi \partial^{\underline{m}} \phi + \frac{5}{2} \partial_{\underline{m}} \phi \partial^{\underline{m}} \rho - \frac{1}{2 \cdot 3!} (H_{(3)})^2 \Big) - \frac{1}{2} e^{\frac{5}{4}\phi + \frac{5}{4}\rho} \left( \partial_{\underline{m}} \chi \partial^{\underline{m}} \chi + \frac{1}{3!} \tilde{F}_{(3)}^2 + \frac{1}{5!} \tilde{G}_{(5)}^2 \right) \\
& \left. \left. + e^{-\frac{5}{4}\phi + \frac{3}{4}\rho} R^{\mathcal{M}_5} \right] \right. \quad (3.2)
\end{aligned}$$

After partial integration, one finds

$$\begin{aligned}
I = & \frac{1}{2\kappa_5^2} \int d^5\hat{x} \sqrt{-h} \left[ e^{-\frac{3}{4}\phi + \frac{5}{4}\rho} \left( R^{(5)} + \frac{5}{4} \partial_{\underline{m}} \rho \partial^{\underline{m}} \rho + \frac{1}{4} \partial_{\underline{m}} \phi \partial^{\underline{m}} \phi - \frac{5}{2} \partial_{\underline{m}} \rho \partial^{\underline{m}} \phi - \frac{1}{2 \cdot 3!} H_{(3)}^2 \right) \right. \\
& \left. - \frac{1}{2} e^{\frac{5}{4}\phi + \frac{5}{4}\rho} \left( \partial_{\underline{m}} \chi \partial^{\underline{m}} \chi + \frac{1}{3!} \tilde{F}_{(3)}^2 + \frac{1}{5!} \tilde{G}_{(5)}^2 \right) + e^{-\frac{5}{4}\phi + \frac{3}{4}\rho} R^{\mathcal{M}_5} \right] \quad (3.3)
\end{aligned}$$

Let us split the index  $\underline{m}$ :

$$\underline{m} = \mu, r \quad (3.4)$$

Carrying out ADM decomposition and adding GH boundary terms yields (see, e.g., [18])

$$\begin{aligned}
\int d^5\hat{x} \mathcal{L}_{bulk+bd} = & \int dr d^4x \sqrt{-g} n \left[ e^{-\frac{3}{4}\phi + \frac{5}{4}\rho} \left( -K_{\mu\nu}^2 + K^2 \right. \right. \\
& - \frac{3}{2n} [\partial_r \phi - n^\mu \partial_\mu \phi] K + \frac{5}{2n} [\partial_r \rho - n^\mu \partial_\mu \rho] K \\
& + \frac{1}{n^2} \left\{ \frac{1}{4} [\partial_r \phi - n^\mu \partial_\mu \phi]^2 + \frac{5}{4} [\partial_r \rho - n^\mu \partial_\mu \rho]^2 \right. \\
& - \frac{5}{2} [\partial_r \phi - n^\mu \partial_\mu \phi] [\partial_r \rho - n^\nu \partial_\nu \rho] - \frac{1}{4} [H_{r\mu\nu} - n^\lambda H_{\lambda\mu\nu}]^2 \Big\} \\
& - \frac{1}{2n^2} e^{\frac{5}{4}\phi + \frac{5}{4}\rho} \left\{ [\partial_r \chi - n^\mu \partial_\mu \chi]^2 + \frac{1}{2} [\tilde{F}_{r\mu\nu} - n^\lambda \tilde{F}_{\lambda\mu\nu}]^2 \right. \\
& \left. \left. + \frac{1}{24} [\tilde{G}_{r\mu\nu\lambda\rho} - n^\sigma \tilde{G}_{\sigma\mu\nu\lambda\rho}]^2 \right\} + \mathcal{L}^{(4)} \right], \quad (3.5)
\end{aligned}$$

where  $r$  is one of the spatial coordinates and will play the role of "time" in the next section, and

$$\begin{aligned}
\mathcal{L}^{(4)} \equiv & e^{-\frac{3}{4}\phi + \frac{5}{4}\rho} \left( R^{(4)} + \frac{3}{2} \nabla_\mu \nabla^\mu \phi - \frac{5}{2} \nabla_\mu \nabla^\mu \rho - \frac{7}{8} \partial_\mu \phi \partial^\mu \phi - \frac{15}{8} \partial_\mu \rho \partial^\mu \rho + \frac{5}{4} \partial_\mu \phi \partial^\mu \rho - \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} \right) \\
& - \frac{1}{2} e^{\frac{5}{4}\phi + \frac{5}{4}\rho} \left( \partial_\mu \chi \partial^\mu \chi + \frac{1}{3!} \tilde{F}^{\mu\nu\rho} \tilde{F}_{\mu\nu\rho} \right) + e^{-\frac{5}{4}\phi + \frac{3}{4}\rho} R^{\mathcal{M}_5} \quad (3.6)
\end{aligned}$$

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<sup>5</sup>In the earlier version of this work, the cross term  $\frac{5}{2} \partial_{\underline{m}} \phi \partial^{\underline{m}} \rho$  was missed. (This has been pointed by Sato and Tsuchiya.)

With  $r$  playing the role of "time", the total "hamiltonian" of the system<sup>6</sup>

$$\mathcal{H} = \pi^{\mu\nu}\partial_r g_{\mu\nu} + \pi_\phi\partial_r\phi + \pi_\rho\partial_r\rho + \pi_B^{\mu\nu}\partial_r B_{\mu\nu} + \pi_\chi\partial_r\chi + \pi_C^{\mu\nu}\partial_r C_{\mu\nu} + \pi_D^{\mu\nu\lambda\rho}\partial_r D_{\mu\nu\lambda\rho} - \mathcal{L}_{bulk+bd} \quad (3.7)$$

can be written as (see, e.g., [6])

$$\mathcal{H} \equiv nH + n_\mu H^\mu + B_{r\mu} Z_B^\mu + C_{r\mu} Z_C^\mu + D_{r\mu\nu\lambda} Z_D^{\mu\nu\lambda} \quad (3.8)$$

Here  $n$ ,  $n_\mu$ ,  $B_{r\mu}$ ,  $C_{r\mu}$  and  $D_{r\mu\nu\lambda}$  behave like Lagrange multipliers, giving the following set of constraints

$$H = 0, \quad H^\mu = 0, \quad Z_B^\mu = 0, \quad Z_C^\mu = 0, \quad Z_D^{\mu\nu\lambda} = 0. \quad (3.9)$$

One can show

$$\begin{aligned} H = & e^{\frac{3}{4}\phi - \frac{5}{4}\rho} \left( -\pi_{\mu\nu}^2 - \frac{1}{2}\pi_\mu^\mu \pi_\phi + \frac{1}{2}\pi_\mu^\mu \pi_\rho - \frac{1}{2}\pi_\phi^2 - \frac{3}{10}\pi_\rho^2 \right. \\ & \left. - (\pi_{B\mu\nu} + \chi\pi_{C\mu\nu} + 6C^{\lambda\rho}\pi_{D\mu\nu\lambda\rho})^2 \right) \\ & - e^{-\frac{5}{4}\phi - \frac{5}{4}\rho} \left( \frac{1}{2}\pi_\chi^2 + \pi_{C\mu\nu}^2 + 12\pi_{D\mu\nu\lambda\rho}^2 \right) - \mathcal{L}^{(4)} \end{aligned} \quad (3.10)$$

and

$$\begin{aligned} H^\mu &= -2\nabla_\nu \pi^{\mu\nu} + \pi_\phi \partial^\mu \phi + \pi_\rho \partial^\mu \rho + \pi_{B\nu\lambda} H^{\mu\nu\lambda} \\ &\quad + \pi_\chi \partial^\mu \chi + \pi_{C\nu\lambda} F^{\mu\nu\lambda} + \pi_{D\nu\lambda\rho\sigma} (G^{\mu\nu\lambda\rho\sigma} - 4C^{\mu\nu} H^{\lambda\rho\sigma}) \\ Z_B^\mu &= 2\nabla_\nu \pi_B^{\mu\nu} \\ Z_C^\mu &= 2\nabla_\nu \pi_C^{\mu\nu} + 4\pi_D^{\mu\nu\lambda\rho} H_{\nu\lambda\rho} \\ Z_D^{\mu\nu\lambda} &= 4\nabla_\rho \pi_D^{\mu\nu\lambda\rho} \end{aligned} \quad (3.11)$$

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<sup>6</sup>We choose the Hamiltonian as follows:

$$S = \int d^5\hat{x} \mathcal{L}_{bulk+bd} = \int dr d^4x \sqrt{-g} (\mathbf{P} \cdot \partial_r \mathbf{Q} - \mathcal{H}),$$

where  $\mathbf{Q}$  and  $\mathbf{P}$  are the "coordinates" and the corresponding "momenta"

$$\mathbf{P} = \frac{1}{\sqrt{-g}} \frac{\delta S}{\delta \partial_r \mathbf{Q}}.$$

Define  $\bar{g}_{\mu\nu}(x, r)$  to be a classical solution to the field equation associated with (3.5) with the following boundary condition<sup>7</sup>

$$\bar{g}_{\mu\nu}(x, r_0) = g_{\mu\nu}(x) \quad , \quad \pi^{\mu\nu}(x) = \bar{\pi}^{\mu\nu}(x, r = r_0) \quad (3.12)$$

The boundary configurations for other fields are similarly defined. The standard procedure of the HJ formalism yields,

$$\pi^{\mu\nu}(x) = \frac{1}{\sqrt{-g(x)}} \frac{\delta S}{\delta g_{\mu\nu}(x)}, \quad (3.13)$$

and similarly for other fields. The HJ equation reads

$$\begin{aligned} & e^{\frac{3}{4}\phi - \frac{5}{4}\rho} \left[ - \left( \frac{1}{\sqrt{-g}} \frac{\delta S_0}{\delta g_{\mu\nu}} \right)^2 - \frac{1}{2} \frac{g_{\mu\nu}}{\sqrt{-g}} \frac{\delta S_0}{\delta g_{\mu\nu}} \frac{1}{\sqrt{-g}} \frac{\delta S_0}{\delta \phi} - \frac{1}{2} \left( \frac{1}{\sqrt{-g}} \frac{\delta S_0}{\delta \phi} \right)^2 \right. \\ & \quad - \frac{3}{10} \left( \frac{1}{\sqrt{-g}} \frac{\delta S_0}{\delta \rho} \right)^2 + \frac{1}{2} \frac{g_{\mu\nu}}{\sqrt{-g}} \frac{\delta S_0}{\delta g_{\mu\nu}} \frac{1}{\sqrt{-g}} \frac{\delta S_0}{\delta \rho} \\ & \quad \left. - \frac{1}{(\sqrt{-g})^2} \left( \frac{\delta S_0}{\delta B_{\mu\nu}} + \chi \frac{\delta S_0}{\delta C_{\mu\nu}} + 6C_{\lambda\rho} \frac{\delta S_0}{\delta D_{\mu\nu\lambda\rho}} \right)^2 \right] \\ & - \frac{e^{-\frac{5}{4}\phi - \frac{5}{4}\rho}}{(\sqrt{-g})^2} \left[ \left( \frac{1}{2} \frac{\delta S_0}{\delta \chi} \right)^2 + \left( \frac{\delta S_0}{\delta C_{\mu\nu}} \right)^2 + 12 \left( \frac{\delta S_0}{\delta D_{\mu\nu\lambda\rho}} \right)^2 \right] \\ & - \left[ e^{-\frac{3}{4}\phi + \frac{5}{4}\rho} \left( R^{(4)} + \frac{3}{2} \nabla_\mu \nabla^\mu \phi - \frac{5}{2} \nabla_\mu \nabla^\mu \rho - \frac{7}{8} \partial_\mu \phi \partial^\mu \phi - \frac{15}{8} \partial_\mu \rho \partial^\mu \rho + \frac{5}{4} \partial_\mu \phi \partial^\mu \rho \right) \right. \\ & \quad \left. - \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} - \frac{1}{2} e^{\frac{5}{4}\phi + \frac{5}{4}\rho} \left( \partial_\mu \chi \partial^\mu \chi + \frac{1}{3!} \tilde{F}^{\mu\nu\rho} \tilde{F}_{\mu\nu\rho} \right) + e^{-\frac{5}{4}\phi + \frac{3}{4}\rho} R^{\mathcal{M}_5} \right] \\ & = 0 \end{aligned} \quad (3.14)$$

The action (3.14) does admit a DBI form of (3.34) once we assume that the fields are constant on the fixed "time" surface as in [6–8]. (In that case, only the  $R^{\mathcal{M}_5}$  term contributes to the HJ equation among the terms in  $\mathcal{L}^{(4)}$ .) We will discuss this in the next subsection. In the remainder of this section, we address the issue of the appearance of the gauge field. Also, we discuss the HJ procedure keeping all the terms in  $\mathcal{L}^{(4)}$ . An interesting toggle

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<sup>7</sup>The  $r = r_0$  surface should not be taken as a genuine boundary. If it were a genuine boundary, the metric on the boundary would be a constant as implied by the Dirichlet boundary condition. (Recall that the GH boundary terms were introduced for Dirichlet boundary conditions for the metric.) Rather it should be taken as a device that bridges the bulk description and the hypersurface description.

between  $S_5$  and  $\mathcal{H}_5$  will be noted depending on whether one uses constant field approximation.

The appearance of a gauge field through a spontaneous symmetry breaking should be a general phenomenon independent of coefficients in  $H$ . Indeed, it is a general phenomenon as proved by the following observation. The solution  $S$  that appears in (3.13) can be viewed as a functional of an antisymmetric "moduli field",  $F_{\mu\nu}$ ,

$$S = S[F_{\mu\nu}] \quad (3.15)$$

To be able to view  $F_{\mu\nu}$  as a field strength of a gauge field, closure of  $F_{\mu\nu}$  must be established. Let us take a covariant derivative on (3.13)

$$0 = \nabla_{[\kappa} \pi^{\mu\nu]}(x) = \nabla_{[\kappa} \frac{1}{\sqrt{-g(x)}} \frac{\delta S}{\delta g_{\mu\nu]}(x)} \quad (3.16)$$

This implies

$$\nabla_{[\kappa} F_{\mu\nu]} = 0, \quad (3.17)$$

and therefore  $F_{\mu\nu}$  can be taken as the field strength of a gauge field,  $A_\mu$ .<sup>8</sup>

For the solution, one can perform the type of derivative expansion considered, e.g., in [10] [12]. Let us set

$$S_0 \equiv S_0^{(0)} + S_0^{(1)} + \dots \quad (3.18)$$

where  $S_0^{(0)}$  ( $S_0^{(1)}$ ) represents the leading (next) order term in the derivative expansion. Let us work out  $S_0^{(0)}$  and  $S_0^{(1)}$ . In the leading order, the HJ equation reads

$$\frac{e^{\frac{3}{4}\phi - \frac{5}{4}\rho}}{(\sqrt{-g})^2} \left[ -\left(\frac{\delta S_0^{(0)}}{\delta g_{\mu\nu}}\right)^2 - \frac{1}{2}g^{\mu\nu} \frac{\delta S_0^{(0)}}{\delta g_{\mu\nu}} \frac{\delta S_0^{(0)}}{\delta \phi} - \frac{1}{2}\left(\frac{\delta S_0^{(0)}}{\delta \phi}\right)^2 \right]$$

---

<sup>8</sup>There are some subtleties here. Strictly speaking,  $S = S[\mathcal{F}_{\mu\nu}]$  with  $\mathcal{F}_{\mu\nu} \equiv -B_{\mu\nu} + F_{\mu\nu}$ . One may simply consider the  $B_{\mu\nu} = 0$  case to avoid unnecessary complications. One can go a little further by the following observation: the right hand side of (3.16) vanishes due to the (anti)symmetry property. Therefore,  $\mathcal{F}_{\mu\nu}$  is closed, and this implies the closure of  $B_{\mu\nu}$ . (Remember that one can consider the solution of HJ equation with the vanishing moduli field  $F_{\mu\nu} = 0$ .) In other words, only a closed  $B_{\mu\nu}$  can be a solution of HJ equation. Another subtlety is a question whether  $F_{\mu\nu}$  would be abelian or non-abelian. We comment on this and related issues in the conclusion.

$$\begin{aligned}
& -\frac{3}{10}\left(\frac{\delta S_0^{(0)}}{\delta \rho}\right)^2 + \frac{1}{2}g_{\mu\nu}\frac{\delta S_0^{(0)}}{\delta g_{\mu\nu}}\frac{\delta S_0^{(0)}}{\delta \rho} \Big] \\
& -\frac{e^{-\frac{5}{4}\phi-\frac{5}{4}\rho}}{(\sqrt{-g})^2}\left[12\left(\frac{\delta S_0^{(0)}}{\delta D_{\mu\nu\lambda\rho}}\right)^2\right] - e^{-\frac{5}{4}\phi+\frac{3}{4}\rho}R^{\mathcal{M}_5} = 0
\end{aligned} \tag{3.19}$$

Although (3.34) is not a solution once the terms with derivatives in  $\mathcal{L}^{(4)}$  are included, the similar types of the terms that would appear when (3.34) is expanded should appear in the solution. Guided by this let us try the following ansatz

$$S_0^{(0)} = \beta_{(0)} \int d^4x \sqrt{-g} e^{-\phi+\rho} \tag{3.20}$$

Substituting (3.20) into (3.19) leads to

$$\frac{1}{5}\beta_{(0)}^2 = R^{\mathcal{M}_5} \tag{3.21}$$

This indicates that one should take  $\mathcal{M}_5 = S_5$ . At the next order, the HJ equation takes

$$\begin{aligned}
& \frac{e^{\frac{3}{4}\phi-\frac{5}{4}\rho}}{(\sqrt{-g})^2} \left[ -2\frac{\delta S_0^{(0)}}{\delta g_{\mu\nu}}\frac{\delta S_0^{(1)}}{\delta g_{\rho\sigma}}g_{\mu\rho}g_{\nu\sigma} - \frac{1}{2}g_{\mu\nu}\frac{\delta S_0^{(0)}}{\delta g_{\mu\nu}}\frac{\delta S_0^{(1)}}{\delta \phi} - \frac{1}{2}g_{\mu\nu}\frac{\delta S_0^{(1)}}{\delta g_{\mu\nu}}\frac{\delta S_0^{(0)}}{\delta \phi} \right. \\
& \quad - \frac{\delta S_0^{(0)}}{\delta \phi}\frac{\delta S_0^{(1)}}{\delta \phi} - \frac{3}{5}\frac{\delta S_0^{(0)}}{\delta \rho}\frac{\delta S_0^{(1)}}{\delta \rho} + \frac{1}{2}g_{\mu\nu}\frac{\delta S_0^{(0)}}{\delta g_{\mu\nu}}\frac{\delta S_0^{(1)}}{\delta \rho} \\
& \quad \left. + \frac{1}{2}g_{\mu\nu}\frac{\delta S_0^{(1)}}{\delta g_{\mu\nu}}\frac{\delta S_0^{(0)}}{\delta \rho} - \left(\frac{\delta S_0^{(1)}}{\delta B_{\mu\nu}} + \chi\frac{\delta S_0^{(1)}}{\delta C_{\mu\nu}} + 6C_{\lambda\rho}\frac{\delta S_0^{(0)}}{\delta D_{\mu\nu\lambda\rho}}\right)^2 \right] \\
& - \frac{e^{-\frac{5}{4}\phi-\frac{5}{4}\rho}}{(\sqrt{-g})^2} \left[ \left(\frac{\delta S_0^{(1)}}{\delta C_{\mu\nu}}\right)^2 \right] - \left[ e^{-\frac{3}{4}\phi+\frac{5}{4}\rho} \left( R^{(4)} + \frac{3}{2}\nabla_\mu\nabla^\mu\phi - \frac{5}{2}\nabla_\mu\nabla^\mu\rho - \frac{7}{8}\partial_\mu\phi\partial^\mu\phi \right. \right. \\
& \quad \left. \left. - \frac{15}{8}\partial_\mu\rho\partial^\mu\rho + \frac{5}{4}\partial_\mu\phi\partial^\mu\rho - \frac{1}{12}H_{\mu\nu\rho}H^{\mu\nu\rho} \right) - \frac{1}{2}e^{\frac{5}{4}\phi+\frac{5}{4}\rho} \left( \partial_\mu\chi\partial^\mu\chi + \frac{1}{3!}\tilde{F}^{\mu\nu\rho}\tilde{F}_{\mu\nu\rho} \right) \right] \\
& = 0
\end{aligned} \tag{3.22}$$

With the zeroth order solution (3.20) substituted, this reduces to

$$\begin{aligned}
& \frac{e^{-\frac{1}{4}\phi-\frac{1}{4}\rho}}{\sqrt{-g}}\beta_{(0)} \left[ \frac{2}{5}\frac{\delta S_0^{(1)}}{\delta \rho} - \frac{e^{\phi-\rho}}{\beta_{(0)}\sqrt{-g}} \left( \frac{\delta S_0^{(1)}}{\delta B_{\mu\nu}} + \chi\frac{\delta S_0^{(1)}}{\delta C_{\mu\nu}} \right)^2 \right] - \frac{e^{-\frac{5}{4}\phi-\frac{5}{4}\rho}}{(\sqrt{-g})^2} \left( \frac{\delta S_0^{(1)}}{\delta C_{\mu\nu}} \right)^2 \\
& - \left[ e^{-\frac{3}{4}\phi+\frac{5}{4}\rho} \left( R^{(4)} + \frac{3}{2}\nabla_\mu\nabla^\mu\phi - \frac{5}{2}\nabla_\mu\nabla^\mu\rho - \frac{7}{8}\partial_\mu\phi\partial^\mu\phi \right. \right. \\
& \quad \left. \left. - \frac{15}{8}\partial_\mu\rho\partial^\mu\rho + \frac{5}{4}\partial_\mu\phi\partial^\mu\rho - \frac{1}{12}H_{\mu\nu\rho}H^{\mu\nu\rho} \right) - \frac{1}{2}e^{\frac{5}{4}\phi+\frac{5}{4}\rho} \left( \partial_\mu\chi\partial^\mu\chi + \frac{1}{3!}\tilde{F}^{\mu\nu\rho}\tilde{F}_{\mu\nu\rho} \right) \right] \\
& = 0
\end{aligned} \tag{3.23}$$

One can show that (3.23) admits the following form of the solution,

$$\begin{aligned}
S_0^{(1)} &= \frac{1}{\beta_{(0)}} \int d^4x \sqrt{-g} e^{-\frac{1}{2}\phi + \frac{3}{2}\rho} \left[ \frac{5}{3} R^{(4)} + \frac{55}{24} (\partial\phi)^2 + \frac{25}{8} (\partial\rho)^2 - \frac{5}{2} (\partial\phi \cdot \partial\rho) \right] \\
&\quad + \frac{5}{6\beta_{(0)}} \int d^4x \sqrt{-g} e^{-\phi + \rho} (\partial\chi)^2 - \frac{1}{10} \beta_{(0)} \int d^4x \sqrt{-g} e^{\frac{3}{2}\phi + \frac{3}{2}\rho} \mathcal{F}_{\mu\nu} \mathcal{F}^{\mu\nu}
\end{aligned} \tag{3.24}$$

It should be possible to find  $S_0^n, n > 1$  in a similar way.

### 3.2 Reduction that leads to DBI form solution to the HJ equation

Let us verify that the HJ equation associated with (3.5) admits a DBI form solution. In order to make our analysis slightly more general, note the following freedom. Suppose we use

$$\tilde{\phi} \rightarrow a\hat{\phi} + b\hat{\rho} \quad \tilde{\rho} \rightarrow c\hat{\phi} + d\hat{\rho} \tag{3.25}$$

in the ansatz (2.2). Then one should get (2.5) where  $\tilde{\phi}, \tilde{\rho}$  are replaced by  $a\hat{\phi} + b\hat{\rho}, c\hat{\phi} + d\hat{\rho}$  respectively. Let us utilize this freedom and introduce the following linear combinations of  $\phi, \rho$ ,

$$\begin{aligned}
\phi &\equiv a\hat{\phi} + b\hat{\rho} \\
\rho &\equiv c\hat{\phi} + d\hat{\rho}
\end{aligned} \tag{3.26}$$

Now Eq.(3.5) can be rewritten

$$\begin{aligned}
&\int d^5\hat{x} \mathcal{L}_{bulk+bd} \\
= &\int dr d^4x \sqrt{-g} n \left[ e^{-\frac{3}{4}(a\hat{\phi}+b\hat{\rho})+\frac{5}{4}(c\hat{\phi}+d\hat{\rho})} \left( -K_{\mu\nu}^2 + K^2 + \frac{u_1}{n} f(\hat{\phi})K + \frac{u_2}{n} f(\hat{\rho})K \right. \right. \\
&\quad \left. \left. + \frac{1}{n^2} \left\{ u_3 f(\hat{\phi})^2 + u_4 f(\hat{\rho})^2 + u_5 f(\hat{\phi})f(\hat{\rho}) - \frac{1}{4} [H_{r\mu\nu} - n^\lambda H_{\lambda\mu\nu}]^2 \right\} \right) \right. \\
&\quad \left. - \frac{1}{2n^2} e^{\frac{5}{4}(a\hat{\phi}+b\hat{\rho})+\frac{5}{4}(c\hat{\phi}+d\hat{\rho})} \left\{ [\partial_r \chi - n^\mu \partial_\mu \chi]^2 + \frac{1}{2} [\tilde{F}_{r\mu\nu} - n^\lambda \tilde{F}_{\lambda\mu\nu}]^2 \right. \right. \\
&\quad \left. \left. + \frac{1}{24} [\tilde{G}_{r\mu\nu\lambda\rho} - n^\sigma \tilde{G}_{\sigma\mu\nu\lambda\rho}]^2 \right\} + \mathcal{L}^{(4)} \right],
\end{aligned} \tag{3.27}$$

where

$$\begin{aligned} f(\hat{\phi}) &\equiv \partial_r \hat{\phi} - n^\mu \partial_\mu \hat{\phi} \\ f(\hat{\rho}) &\equiv \partial_r \hat{\rho} - n^\mu \partial_\mu \hat{\rho} \end{aligned} \quad (3.28)$$

and the  $u$ 's are related to  $a, b, c, d$

$$\begin{aligned} u_1 &\equiv -\frac{3}{2}a + \frac{5}{2}c, & u_2 &\equiv -\frac{3}{2}b + \frac{5}{2}d, & u_3 &\equiv \frac{1}{4}a^2 + \frac{5}{4}c^2 - \frac{5}{2}ac, \\ u_4 &\equiv \frac{1}{4}b^2 + \frac{5}{4}d^2 - \frac{5}{2}bd, & u_5 &\equiv \frac{1}{2}ab + \frac{5}{2}cd - \frac{5}{2}bc - \frac{5}{2}ad \end{aligned} \quad (3.29)$$

After some algebra, one can show

$$\begin{aligned} H &= e^{\frac{3}{4}(a\hat{\phi}+b\hat{\rho})-\frac{5}{4}(c\hat{\phi}+d\hat{\rho})} \left( -\pi_{\mu\nu}^2 + w_1(\pi_\mu^\mu)^2 + w_2\pi_{\hat{\rho}}^2 + w_3\pi_{\hat{\phi}}^2 + w_4\pi_\mu^\mu\pi_{\hat{\phi}} + w_5\pi_\mu^\mu\pi_{\hat{\rho}} + w_6\pi_{\hat{\phi}}\pi_{\hat{\rho}} \right. \\ &\quad \left. -(\pi_{B\mu\nu} + \chi\pi_{C\mu\nu} + 6C^{\lambda\rho}\pi_{D\mu\nu\lambda\rho})^2 \right) \\ &\quad -e^{-\frac{5}{4}(a\hat{\phi}+b\hat{\rho})-\frac{5}{4}(c\hat{\phi}+d\hat{\rho})} \left( \frac{1}{2}\pi_\chi^2 + \pi_{C\mu\nu}^2 + 12\pi_{D\mu\nu\lambda\rho}^2 \right) - \mathcal{L}^{(4)} \end{aligned} \quad (3.30)$$

The parameters  $w_1, \dots, w_6$  are related to  $u$ 's that appear in (3.29) by

$$\begin{aligned} w_1 &\equiv \frac{u_2^2 u_3 + u_1^2 u_4 - 4u_3 u_4 - u_1 u_2 u_5 + u_5^2}{D} \\ w_2 &\equiv \frac{u_1^2 - 3u_3}{D} & w_3 &\equiv \frac{u_2^2 - 3u_4}{D} \\ w_4 &\equiv \frac{2u_1 u_4 - u_2 u_5}{D} & w_5 &\equiv \frac{2u_2 u_3 - u_1 u_5}{D} & w_6 &\equiv -\frac{2u_1 u_2 - 3u_5}{D} \end{aligned} \quad (3.31)$$

with

$$D \equiv 4u_2^2 u_3 + 4u_1^2 u_4 - 12u_3 u_4 - 4u_1 u_2 u_5 + 3u_5^2 \quad (3.32)$$

As in [6–8], we assume that the fields are constant on the fixed "time" surface. Due to this assumption, only the  $R^{\mathcal{M}_5}$  term contributes to the HJ equation among the terms in  $\mathcal{L}^{(4)}$ ; substituting (3.13) into the hamiltonian constraint, one finds the following HJ equation:

$$e^{\frac{3}{4}(a\hat{\phi}+b\hat{\rho})-\frac{5}{4}(c\hat{\phi}+d\hat{\rho})} \left[ -\left( \frac{1}{\sqrt{-g}} \frac{\delta S_0}{\delta g_{\mu\nu}} \right)^2 + w_1 \left( \frac{g_{\mu\nu}}{\sqrt{-g}} \frac{\delta S_0}{\delta g_{\mu\nu}} \right)^2 + w_4 \frac{g_{\mu\nu}}{\sqrt{-g}} \frac{\delta S_0}{\delta g_{\mu\nu}} \frac{1}{\sqrt{-g}} \frac{\delta S_0}{\delta \hat{\phi}} \right]$$

$$\begin{aligned}
& +w_3\left(\frac{1}{\sqrt{-g}}\frac{\delta S_0}{\delta\hat{\phi}}\right)^2 + w_2\left(\frac{1}{\sqrt{-g}}\frac{\delta S_0}{\delta\hat{\rho}}\right)^2 + w_6\frac{1}{\sqrt{-g}}\frac{\delta S_0}{\delta\hat{\phi}}\frac{1}{\sqrt{-g}}\frac{\delta S_0}{\delta\hat{\rho}} + w_5\frac{g_{\mu\nu}}{\sqrt{-g}}\frac{\delta S_0}{\delta g_{\mu\nu}}\frac{1}{\sqrt{-g}}\frac{\delta S_0}{\delta\hat{\rho}} \\
& \quad - \left(\frac{1}{\sqrt{-g}}\frac{\delta S_0}{\delta B_{\mu\nu}} + \chi\frac{1}{\sqrt{-g}}\frac{\delta S_0}{\delta C_{\mu\nu}} + 6C_{\lambda\rho}\frac{1}{\sqrt{-g}}\frac{\delta S_0}{\delta D_{\mu\nu\lambda\rho}}\right)^2 \Big] \\
& - e^{-\frac{5}{4}(a\hat{\phi}+b\hat{\rho})-\frac{5}{4}(c\hat{\phi}+d\hat{\rho})} \left[ \left(\frac{1}{2}\frac{1}{\sqrt{-g}}\frac{\delta S_0}{\delta\chi}\right)^2 + \left(\frac{1}{\sqrt{-g}}\frac{\delta S_0}{\delta C_{\mu\nu}}\right)^2 + 12\left(\frac{1}{\sqrt{-g}}\frac{\delta S_0}{\delta D_{\mu\nu\lambda\rho}}\right)^2 \right] \\
& = e^{-\frac{5}{4}(a\hat{\phi}+b\hat{\rho})+\frac{3}{4}(c\hat{\phi}+d\hat{\rho})} R^{\mathcal{M}^5}
\end{aligned} \tag{3.33}$$

Following [6], let us examine whether (3.33) admits the following form of the solution, which is a slight modification of the corresponding solution in [6]:

$$S_0 = S_c + S_{DBI} + S_{WZ} \tag{3.34}$$

where

$$S_c = \alpha \int d^4x \sqrt{-g} e^{\zeta_3\hat{\phi}+\zeta_4\hat{\rho}} \tag{3.35}$$

$$S_{DBI} = \beta \int d^4x e^{\zeta_1\hat{\phi}+\zeta_2\hat{\rho}} \sqrt{-\det(g_{\mu\nu} + \mathcal{F}_{\mu\nu})} \tag{3.36}$$

$$S_{WZ} = \gamma \int d^4x \varepsilon^{\mu\nu\lambda\rho} \left( \frac{1}{24} D_{\mu\nu\lambda\rho} + \frac{1}{4} C_{\mu\nu} \mathcal{F}_{\lambda\rho} + \frac{1}{8} \chi \mathcal{F}_{\mu\nu} \mathcal{F}_{\lambda\rho} \right), \tag{3.37}$$

where

$$\mathcal{F}_{\mu\nu} \equiv -B_{\mu\nu} + F_{\mu\nu} \tag{3.38}$$

Inspection of the terms' structures reveals that the presence of  $(\pi_\mu^\mu)^2$  would require a major modification of (3.34); let us impose

$$w_1 = 0 \tag{3.39}$$

Detailed computation implies that (3.33) would admit a solution of the form (3.34) once the following conditions are imposed in addition to the previous condition (3.39):

$$\begin{aligned}
2w_4\zeta_1 + 2w_5\zeta_2 &= 1, \quad \beta^2(w_3\zeta_1^2 + w_2\zeta_2^2 + w_6\zeta_1\zeta_2) = -\frac{1}{2}\gamma^2, \\
\zeta_1 &= -a, \quad \zeta_2 = -b, \quad \zeta_3 = -a + c, \quad \zeta_4 = -b + d \\
\alpha^2(-1 + 2w_4\zeta_3 + w_3\zeta_3^2 + w_2\zeta_4^2 + w_6\zeta_3\zeta_4 + 2w_5\zeta_4) &= R^{\mathcal{M}^5}, \\
w_4\zeta_3 + w_5\zeta_4 &= 1, \quad 2w_4\zeta_1 + 2w_3\zeta_1\zeta_3 + 2w_2\zeta_2\zeta_4 + w_6\zeta_2\zeta_3 + w_6\zeta_1\zeta_4 + 2w_5\zeta_2 = 0
\end{aligned} \tag{3.40}$$



The constraints (3.39) and (3.40) amount to 4 constraints among  $a, b, c$  and  $d$ . In other words, one can first use the second line to replace  $\zeta$ 's by the corresponding expressions on the right-hand sides of the second line. One can then solve

$$\begin{aligned} w_1 = 0, \quad 2w_4\zeta_1 + 2w_5\zeta_2 = 1, \quad w_4\zeta_3 + w_5\zeta_4 = 1 \\ 2w_4\zeta_1 + 2w_3\zeta_1\zeta_3 + 2w_2\zeta_2\zeta_4 + w_6\zeta_2\zeta_3 + w_6\zeta_1\zeta_4 + 2w_5\zeta_2 = 0 \end{aligned} \quad (3.41)$$

where  $\zeta$ 's should take the explicit expression in terms of  $(a, b, c, d)$ . Once the solutions are determined, they can be substituted into the remaining two equations

$$\begin{aligned} \beta^2(w_3\zeta_1^2 + w_2\zeta_2^2 + w_6\zeta_1\zeta_2) = -\frac{1}{2}\gamma^2 \\ \alpha^2(-1 + 2w_4\zeta_3 + w_3\zeta_3^2 + w_2\zeta_4^2 + w_6\zeta_3\zeta_4 + 2w_5\zeta_4) = R^{\mathcal{M}_5} \end{aligned} \quad (3.42)$$

and these equations will determine the relations between  $\alpha, \beta, \gamma$ . Interestingly, it turns out that the four equations (3.41) are automatically satisfied. This implies that one can freely choose  $(a, b, c, d)$ ; as far as the remaining equations in (3.40),

$$\begin{aligned} \beta^2(w_3\zeta_1^2 + w_2\zeta_2^2 + w_6\zeta_1\zeta_2) = -\frac{1}{2}\gamma^2 \\ \zeta_1 = -a, \quad \zeta_2 = -b, \quad \zeta_3 = -a + c, \quad \zeta_4 = -b + d \\ \alpha^2(-1 + 2w_4\zeta_3 + w_3\zeta_3^2 + w_2\zeta_4^2 + w_6\zeta_3\zeta_4 + 2w_5\zeta_4) = R^{\mathcal{M}_5}, \end{aligned} \quad (3.43)$$

are satisfied, the reduction will be consistent. The first and third equation in (3.43) becomes

$$\begin{aligned} \beta^2 &= \gamma^2 \\ \frac{1}{5}\alpha^2 &= R^{\mathcal{M}_5} \end{aligned} \quad (3.44)$$

regardless of values of  $(a, b, c, d)$ , and therefore one must take

$$\mathcal{M}_5 = S_5 \quad (3.45)$$

The case we have considered in the previous subsection corresponds to<sup>9</sup>

$$a = d = 1, \quad b = c = 0 \quad (3.48)$$

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<sup>9</sup>One can also consider the case

$$a = \frac{8}{3}, \quad d = 1, \quad b = c = 0 \quad (3.46)$$

which leads to

$$u_1 = -\frac{3}{2}, \quad u_2 = \frac{5}{2}, \quad u_3 = \frac{1}{4}, \quad u_4 = \frac{5}{4}, \quad u_5 = -\frac{5}{2} \quad (3.49)$$

and

$$w_1 = 0, \quad w_2 = -\frac{3}{10}, \quad w_3 = -\frac{1}{2}, \quad w_4 = -\frac{1}{2}, \quad w_5 = \frac{1}{2}, \quad w_6 = 0 \quad (3.50)$$

### 3.3 Implications

In the previous subsection, a gauge theory action was obtained after the HJ procedure. The HJ principal function  $S$  is nothing but the lagrangian with  $r$  playing the role of time. As mentioned in the introduction, the gauge action can be interpreted as a dual action to the 4D gravity action. The 4D gravity system itself is a (gauged) dS supergravity.

#### 3.3.1 on realization of "braneworld"

Let us ponder whether the "braneworld" is realized by the current procedure. First of all, we should note that the current procedure implies a qualitatively different braneworld from the conventional Randall-Sundrum type in that the only dynamical degrees of freedom are those of the gauge multiplet after integrating out the gravitational degrees of freedom.

The situation is analogous to the usual QFT procedure where instantons become dynamical degrees of freedom that are "dual" to the original gauge theory. One has instanton moduli and fluctuation degrees of freedom in the path integral once one expands around an instanton solution. After one integrates out the fluctuation degrees of freedom, one finds an instanton action that can be viewed as "dual" to the original action (See [19] and [2])

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This choice casts the exponential factors in (3.27) into the forms that were considered in [6]. One finds, in this case,

$$u_1 = -4, \quad u_2 = \frac{5}{2}, \quad u_3 = \frac{16}{9}, \quad u_4 = \frac{5}{4}, \quad u_5 = -\frac{20}{3} \quad (3.47)$$

Some of these coefficients are different from those that appeared in [6], and should be an indication that (2.2) and (2.3) belong to a different class of ansatze than those of [6]. Even though the two ansatze are different, they admit the same DBI solutions; we take this as certain robustness of the DBI form solution.

for related discussions.) The HJ procedure is a solution-finding procedure, and we saw the moduli field  $F_{\mu\nu}$  enter for the case at hand. One should then integrate out the 4D gravitational degrees of freedom (i.e., all the other degrees of freedom than the moduli field), and eventually find an action of the moduli field. There will also be the gravitational part of the background, therefore, the ultimate action of the moduli fields would be in that gravity background.<sup>10</sup>

For the braneworld realization, it would be required to check whether a brane solution of (3.34) localizes at some value of  $r$ . As stated above, the current procedure leads to a qualitatively different braneworld. There still exists a feature within the current setup that might be an indication of the localization of all the degrees of freedom:

$$\frac{\partial S}{\partial r_0} = 0. \quad (3.51)$$

### 3.3.2 new paradigm for black hole physics

As discussed in [21], the black hole information paradox is an amenable problem in string theory context. It is in the 4D pure Einstein gravity where the paradox becomes more subtle. The present work may have an application in black hole physics; in particular, in the aspect associated with the information paradox in the 4D pure Einstein gravity.

In the usual approach of QFT in a curved spacetime, the geometry enters as a background whereas the matter fields are treated on the quantum level. It is almost evident that geometry as a non-dynamic background would be inadequate for describing physics in which the back-reaction plays a crucial role. The information paradox should lie in the classical treatment of the geometry (see, e.g., the recent discussion in [22]). The geometry is strictly classical in the conventional approach because the matter quantum fields in the usual approach do not directly describe the fluctuations of the *geometry*.<sup>11</sup>

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<sup>10</sup>The action in the curved background might be identified as including quantum and non-perturbative effects. Such an identification was made in [20] for example.

<sup>11</sup>It is the matter fields that represent the fluctuating degrees of freedom in the conventional approach, and unlike the current ADM reduction approach, the matter fields are extrinsic to the geometric degrees of freedom. One may say that the matter fields indirectly describe the fluctuations of the geometry since they are coupled to the metric. However, this would be so only when the geometry degrees of freedom are quantized as well.

The best solution for this status of matter would be the full quantization of the Einstein-Hilbert action. Given the unavailability of such an apparatus, the second best solution would be to have a *semi-classical* treatment of geometry. The dual gauge action obtained through the ADM reduction of this paper should provide the needed semi-classical tool.

In the usual approach, the matter fields are not intrinsically gravitational degrees of freedom. In contrast, the gauge action obtained as a result of ADM reduction provides degrees of freedom that are intrinsic to the original gravity system. The matter field equations are solved in some background metric in the conventional approach. Since it is not the proper full coupled equations between matter and metric that are solved, the result is bound to be without a proper account of back-reaction from the metric that gets deformed by the matter. In the proposed ADM reduction approach, one gets the "matter" system, i.e., the YM field after the spontaneous symmetry breaking. In other words, the appearance of the "matter fields" is built into the formulation. One can then try to solve those equations associated with YM field. However, the interpretation is now very different: the gauge field equations directly, although semi-classically, describe the fluctuations of the geometry. We will have more on this as well as other speculative issues in the conclusion.

## 4 Domain-wall solution, toroidal compactification and "inflaton"

In this section, we analyze two more aspects of the 5D action (2.5) that has been obtained by the sphere reduction in sec 2. One thing to note is that even if we are using the notation  $r$ , it is not necessarily a radial coordinate; it is one of the spatial coordinates.

### 4.1 Domain-wall solution

One may use either a 5D Einstein-type frame or a "string"-type frame to find a solution. In this section, we use an Einstein type frame. (It should be possible to find the corresponding solution in a 5D "string-type" frame.)

Consider (2.11) which we quote here for convenience,

$$I_5 = \frac{1}{2\kappa_5^2} \int d^5\xi \sqrt{-h} \left[ R^{(5)} - \frac{5}{6}(\partial\rho)^2 - \frac{1}{2}(\partial\phi)^2 - \frac{1}{2 \cdot 3!} e^{-\phi} e^{\frac{5}{3}\rho} H_{(3)}^2 \right. \\ \left. - \frac{1}{2} e^{2\phi} (\partial\chi)^2 - \frac{1}{2 \cdot 3!} e^{\phi} e^{\frac{5}{3}\rho} \tilde{F}_{(3)}^2 - \frac{1}{2 \cdot 5!} e^{\frac{10}{3}\rho} \tilde{G}_{(5)}^2 + e^{-\frac{4}{3}\rho} R^{\mathcal{M}_5} \right] \quad (4.1)$$

and the reduced field equations setting  $\chi = H = \phi = F = 0$ . In this section, we take

$$R^{\mathcal{M}_5} = R^{S^5} \quad (4.2)$$

Below we will set  $G = 0$  as well because only that case admits a relatively simple solution. It follows from the action given in (2.11) with  $F = \chi = H = \phi = 0$  that

$$\nabla_{\underline{m}} \left( e^{\frac{10}{3}\rho} \tilde{G}_{(5)}^{\underline{mn}_1 \dots \underline{n}_4} \right) = 0, \quad (4.3)$$

$$\nabla^2 \rho - \frac{1}{120} e^{\frac{10}{3}\rho} \tilde{G}_{(5)}^2 - \frac{4}{5} e^{-\frac{4}{3}\rho} R^{\mathcal{M}_5} = 0 \quad (4.4)$$

$$R_{\underline{mn}}^{(5)} - \frac{5}{6} \partial_{\underline{m}} \rho \partial_{\underline{n}} \rho - \frac{1}{4 \cdot 4!} e^{\frac{10}{3}\rho} \tilde{G}_{\underline{mpqrs}} \tilde{G}_{\underline{n}}^{\underline{pqrs}} - h_{\underline{mn}} \left( \frac{1}{2} R^{(5)} - \frac{5}{12} (\partial\rho)^2 + \frac{1}{2} e^{-\frac{4}{3}\rho} R^{\mathcal{M}_5} \right) = 0 \quad (4.5)$$

Let us try the following metric ansatz,

$$ds_5^2 = e^{2A} dr^2 + e^{2C(r)} ds_{dS_4}^2 \quad (4.6)$$

The  $G_5$  field equation (4.3) implies

$$G_5^{m_1 \dots m_5} = \frac{k}{\sqrt{-h}} e^{-\frac{10}{3}\rho} \epsilon^{m_1 \dots m_5} \\ G_5^2 = -5! k^2 e^{-\frac{20}{3}\rho} \quad (4.7)$$

$$\tilde{G}_{\underline{mn}_1 \dots \underline{n}_4} \tilde{G}_{\underline{k}}^{\underline{n}_1 \dots \underline{n}_4} = \frac{1}{5} h_{\underline{mk}} \tilde{G}_{(5)}^2. \quad (4.8)$$

Consider  $(rr)$  and  $(11)$  components of (4.5):

$$R^{(5)} - \frac{5}{6} h^{rr} (\partial_r \rho) (\partial_r \rho) + e^{-\frac{4}{3}\rho} R^{\mathcal{M}_5} - \frac{2}{h_{rr}} \left[ R_{rr}^{(5)} - \frac{5}{6} (\partial_r \rho) (\partial_r \rho) - \frac{1}{4 \cdot 4!} e^{\frac{10}{3}\rho} \tilde{G}_{r\underline{pqrs}} \tilde{G}_r^{\underline{pqrs}} \right] = 0 \quad (4.9)$$

$$R^{(5)} - \frac{5}{6}h^{rr}(\partial_r\rho)(\partial_r\rho) + e^{-\frac{4}{3}\rho}R^{\mathcal{M}_5} - \frac{2}{h_{11}}\left[R_{11}^{(5)} - \frac{1}{4 \cdot 4!}e^{\frac{10}{3}\rho}\tilde{G}_{1\underline{pqrs}}\tilde{G}_1^{\underline{pqrs}}\right] = 0 \quad (4.10)$$

Combining (4.9) and (4.10), one gets

$$\frac{1}{h_{rr}}\left[R_{rr}^{(5)} - \frac{5}{6}\partial_r\rho\partial_r\rho\right] = \frac{1}{h_{11}}R_{11}^{(5)} \quad (4.11)$$

Using the result, e.g., in appendix B of [16], one can show

$$\begin{aligned} R_{11}^{(5)} &= R_{11}^{(4)} + g_{11}e^{2C-2A}(-4\nabla_r C \nabla_r C - \nabla_r \nabla_r C + \nabla_r A \nabla_r C) \\ R_{rr}^{(5)} &= 4(\nabla_r A \nabla_r C - \nabla_r C \nabla_r C - \nabla_r \nabla_r C) \end{aligned} \quad (4.12)$$

With these, (4.11) yields<sup>12</sup>

$$\begin{aligned} &4\nabla_r A \nabla_r C - 4\nabla_r C \nabla_r C - 4\nabla_r \nabla_r C - \frac{5}{6}(\partial_r\rho)^2 \\ = &e^{2A-2C}(\Lambda - 4e^{2C-2A}\nabla_r C \nabla_r C - e^{2C-2A}\nabla_r \nabla_r C + e^{2C-2A}\nabla_r A \nabla_r C) \end{aligned} \quad (4.14)$$

Let us take  $h^{mn}$  on (4.5):

$$R^{(5)} - \frac{5}{6}h^{rr}(\partial_r\rho)(\partial_r\rho) + \frac{5}{3}e^{-\frac{4}{3}\rho}R^{\mathcal{M}_5} + \frac{1}{6 \cdot 4!}e^{\frac{10}{3}\rho}G_5^2 = 0 \quad (4.15)$$

Combining (4.15) and (4.10) and setting  $G = 0$ <sup>13</sup>, one gets

$$-\frac{2}{3}e^{-\frac{4}{3}\rho}R^{\mathcal{M}_5} = 2e^{-2C}(\Lambda^{(4)} - 4e^{2C}\nabla_r C \nabla_r C - e^{2C}\nabla_r \nabla_r C) \quad (4.16)$$

Then (4.16) implies

$$C = \frac{2}{3}\rho \quad , \quad \Lambda^{(4)} = 3q_2^2 - \frac{1}{3}R^{\mathcal{M}_5} \quad (4.17)$$

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<sup>12</sup>where

$$\nabla_r \nabla_r = \nabla^2 = e^{-A}\partial_r(e^{-A}\partial_r) \quad (4.13)$$

<sup>13</sup>If one keeps  $G$ , two different types of  $e^\rho$  factors are present, and this eliminates possibility of any simple solution.

The  $\rho$ -eq (4.4) takes

$$\frac{d^2\rho}{dr^2} + \left(4\frac{dC}{dr} - \frac{dA}{dr}\right)\frac{d\rho}{dr} - \frac{4}{5}e^{2A-\frac{4}{3}\rho}R^{\mathcal{M}_5} = 0 \quad (4.18)$$

In the absence of  $G_{(5)}$ , one can show that (4.10) takes

$$R^{(5)} - \frac{5}{6}e^{-2A}\left(\frac{d\rho}{dr}\right)^2 + e^{-\frac{4}{3}\rho}R^{\mathcal{M}_5} = \frac{2}{h_{11}}R_{11}^{(5)} \quad (4.19)$$

Let us consider the following set of ansatze:

$$\begin{aligned} A &= 0 \\ \rho &= p \ln(q_1 + q_2 r) \\ C &= \frac{2}{3}p \ln(q_1 + q_2 r) \end{aligned} \quad (4.20)$$

Using

$$R_{(5)} = e^{-2C}R_{(4)} + 4\left[-2\nabla_r\nabla_r C - 5\nabla_r C\nabla_r C\right] \quad (4.21)$$

(4.19) takes

$$\begin{aligned} &e^{-2C}R^{(4)} - 4(2\nabla_r\nabla_r C + 5\nabla_r C\nabla_r C) + e^{-\frac{4}{3}\rho}R^{\mathcal{M}_5} - \frac{5}{6}e^{-2C}(\partial_r\rho)^2 \\ &= 2e^{-2C}(\Lambda - 4e^{2C}\nabla_r C\nabla_r C - e^{2C}\nabla_r\nabla_r C) \end{aligned} \quad (4.22)$$

Substituting (4.20) into (4.18) and (4.14), one can see that  $p = \frac{3}{2}$  and

$$\begin{aligned} R^{\mathcal{M}_5} &= \frac{45}{8}q_2^2 \quad \text{from (4.18)} \\ \Lambda &= \frac{9}{8}q_2^2 \quad \text{from (4.14)} \end{aligned} \quad (4.23)$$

where  $\Lambda (\equiv \Lambda^{(4)})$  is defined by  $R_{\mu\nu}^{(4)} = \Lambda g_{\mu\nu}$ . Eq.(4.22) also produces a consistent result:

$$2\Lambda + R^{\mathcal{M}_5} = \frac{63}{8}q_2^2 \quad (4.24)$$

## 4.2 Toroidal compactification and "inflaton"

Carrying out a conventional dimensional reduction on the 5D action (2.11) will be worthwhile because it will yield a 4D gravity theory with various gauge fields with positive cosmological constant. The theory has been obtained from IIB supergravity, and provides a potentially interesting inflationary model. Let us consider the  $\rho$ -rescaled form (2.9). One can easily carry out dimensional reduction keeping as many fields in (2.9) as one wished. Focusing on the perspective of 4D inflatonary physics, we illustrate the case with the metric and  $\rho$ :

$$I = \frac{1}{2\kappa_5^2} \int d^5\xi \sqrt{-h} \left[ R^{(5)} - \frac{1}{2}(\partial\rho)^2 + e^{-\frac{4}{\sqrt{15}}\rho} R^{\mathcal{M}^5} \right] \quad (4.25)$$

One can consider a simple dimensional reduction given by

$$\rho = \rho(x^\mu) \quad , \quad ds_5^2 = dr^2 + g_{\mu\nu} dx^\mu dx^\nu \quad (4.26)$$

The resulting 4D action takes

$$I = \int d^4x \sqrt{-g} \left[ R^{(4)} - \frac{1}{2}(\partial\rho)^2 + e^{-\frac{4}{\sqrt{15}}\rho} R^{\mathcal{M}^5} \right] \quad (4.27)$$

The gauge fields  $\chi, C, D$  and  $B$  can easily be accommodated. It would be interesting to investigate whether one could construct a dS solution with addition of various form fields. If so, that would be in line with the observation made in the KKLT type approaches. We leave the resulting model's phenomenological study for the future.

## 5 Conclusion

In sec 2, we have carried out reduction of IIB supergravity on  $\mathcal{M}_5 (= \mathcal{H}_5 \text{ or } S_5)$ . The resulting 5D gravity lagrangian has been analyzed in several different directions. In one direction, we have shown that it admits a 4D curved domain-wall solution. In another direction, we have performed, following [6–8], the Hamilton-Jacobi procedure of canonical transformation and have obtained another gravity description. As shown in sec 3, the Hamilton-Jacobi equation admits a class of solutions that take a form of a gauge theory action.<sup>14</sup>

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<sup>14</sup>The fact that the original lagrangian admits a domain-wall solution and its Hamilton-Jacobi equation admits a worldvolume action as a solution should be related although we will not pursue this aspect on a deeper level than is apparent.



The way the gauge field strength  $F_{\mu\nu}$  appears is intriguing. The world-volume gauge fields emerge as "moduli fields": regardless of the values that the gauge fields take, the gauge action satisfies the Hamilton-Jacobi equation of the gravity system, and the gauge fields describe the fluctuations of the moduli space. They must be an inequivalent set of extremum solutions, and the inequivalence must stem from different patterns of the brane fluctuations. Those patterns are parameterized through the field  $F_{\mu\nu}$ . Interestingly, the appearance of the dual degrees of freedom as a form of moduli fields was observed before in the context of forward duality: the strong coupling limit of a DBI action admits a class of solutions that can be collectively interpreted as a closed string action [23–26].

One of the potentially powerful implications is the fact that getting non-gravitational degrees of freedom through ADM reduction would work for the pure 4D Einstein-Hilbert action. We believe that for the proper treatment of the black hole information two conditions are required for the QFT tool adopted to tackle the paradox. The first is that the adopted QFT should *directly* describe the fluctuations of the geometry. It is necessary to use a formulation that is self-consistent or "closed" under the forward and backward dualizations. The second is that the QFT interactions in their precise forms must be included. The ADM reduction would, in principle at least, determine the precise form of the interactions of the resulting gauge theory. In these regards, the ADM reduction approach should provide a proper paradigm for black hole physics.

There are multiple future directions:

One is an obvious direction of studying the supersymmetry aspect of the 5D/4D theories. Several other directions are associated with a better understanding of the ADM reduction itself. One may try to extend the program of sec 3.1 to higher orders in the derivative expansion. Another direction would be to address the following question: what at the full string theory level would be responsible for the appearance of a gauge field from a gravity system? The appearance of a gauge field seems to be a general phenomenon that occurs in a low energy theory that may not have embedding in a string theory. However, it would be still interesting to see the full stringy mechanism that is behind for the theories that do have stringy embedding. (See our speculation below.)

There are other related issues that require further study. One of them is

the issue of whether the emerging gauge field would be abelian or non-abelian. The appearance of an abelian gauge moduli field is straightforward. The real question is if there could be a (relatively simple) way to introduce non-abelian degrees of freedom. At this point we can only state what we anticipate and should postpone a better answer until further research. Presumably, the abelian vs non-abelian issue would depend on whether one uses a collection of D3 branes or, say D1 branes to describe the bulk physics. As observed, e.g., in [1], a higher dimensional abelian brane can be described by a non-abelian lower dimensional branes.

As stated in sec 2, applications of the ADM reduction approach to black hole information should be interesting as well. The other directions concern phenomenological aspects and applications. One may take (4.27) with other form fields as a starting point, and study the resulting 4D Friedmann-Lemaitre-Robertson-Walker eqs in the presence of D3 (or even D7). It will be interesting to make a connection this way with the KKLT and related compactification scenarios.

Finally remarks on more speculative aspects are in order: The *forward dualization* mentioned in the introduction should be associated with endpoints of an open string sticking together and becoming a closed string. By the same token, the appearance of gauge degrees of freedom should presumably be associated with a closed string opens up and becomes an open string on the closed string theory level. It would be very interesting if one could make this more precise and quantitative.

The following question was raised in [27]. The DBI action contains all  $\alpha'$ -order terms<sup>15</sup> but it still appears as a solution of reduced supergravity that is just the leading  $\alpha'$  action of a closed string. Perhaps the answer lies in the following. The gauge form solution represents excitations of massless open string modes. The higher  $\alpha'$  corrections to the IIB supergravity may be associated with a massive gauge action that has all the *massive open string modes* appearing explicitly at first and then subsequently integrated out in the open string context, therefore, deforming the massless gauge field. (The integrating out procedure should be done using the full string theory setup which of course would be a hard step in practice.) Differently put, the massless closed strings viewed as a composite open string state should

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<sup>15</sup>Of course, this is true only in the leading derivative expansion in  $\mathcal{F}_{\mu\nu}$ ; once the sub-leading terms such as  $\partial\mathcal{F}$ ,  $\partial\partial\mathcal{F}$ , etc. are taken into account, new terms would appear.

be massive, and apparently they seem sufficient to account for the massless gauge theory modes.

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## Appendix A: Differential form conventions

We use the following conventions on differential forms. The flat space metric signature is mostly positive, so  $\det \eta_{ab} = -1$  in any space-time dimensions. The Levi-Civita tensor  $\epsilon^{a_1 \dots a_D}$  is defined such that

$$\epsilon^{01 \dots D-1} = 1, \quad \epsilon_{01 \dots D-1} = -1, \quad (\text{A.1})$$

hence

$$\epsilon^{a_1 \dots a_D} \epsilon_{a_1 \dots a_D} = \epsilon_{a_1 \dots a_D} \eta^{a_1 a'_1} \dots \eta^{a_D a'_D} \epsilon_{a'_1 \dots a'_D} = \det \eta \cdot D! = -D! \quad (\text{A.2})$$

Generalization of (A.2) to a curve background is given by

$$\epsilon^{m_1 \dots m_D} \epsilon_{m_1 \dots m_D} = \det g \cdot D! \quad (\text{A.3})$$

For a  $p$ -form we choose

$$\Omega_{(p)} = \frac{1}{p!} \Omega_{m_1 \dots m_p} dx^{m_1} \wedge \dots \wedge dx^{m_p}, \quad (\text{A.4})$$

and

$$\begin{aligned} d\Omega_{(p)} &= \frac{1}{p!} \partial_{[m_{p+1}} \Omega_{m_1 \dots m_p]} dx^{m_{p+1}} \wedge dx^{m_1} \wedge \dots \wedge dx^{m_p} \\ &= \frac{1}{p!} \partial_{[m_1} \Omega_{m_2 \dots m_{p+1}]} dx^{m_1} \wedge dx^{m_2} \wedge \dots \wedge dx^{m_{p+1}}. \end{aligned} \quad (\text{A.5})$$

The external derivative  $d$  acts from the left, i.e.,

$$d(\Omega_{(p)} \wedge \Omega_{(q)}) = d\Omega_{(p)} \wedge \Omega_{(q)} + (-)^p \Omega_{(p)} \wedge d\Omega_{(q)}. \quad (\text{A.6})$$

We define the Hodge star as

$$*(dx^{n_1} \wedge \dots \wedge dx^{n_p}) = \frac{1}{(D-p)!} \frac{1}{\sqrt{-g}} \epsilon_{m_1 \dots m_{D-p}}^{n_1 \dots n_p} dx^{m_1} \wedge \dots \wedge dx^{m_{D-p}}, \quad (\text{A.7})$$

so the dual to  $\Omega_{(p)}$  form  $*\Omega_{(p)}$  is defined by

$$*\Omega_{(p)} = \frac{1}{(D-p)!p!} \frac{1}{\sqrt{-g}} \epsilon_{m_1 \dots m_{D-p}}^{n_1 \dots n_p} \Omega_{n_1 \dots n_p} dx^{m_1} \wedge \dots \wedge dx^{m_{D-p}}. \quad (\text{A.8})$$

On account of the latter expression and eq. (A.3), one gets

$$*_p^2 = (-)^{Dp+p+1}. \quad (\text{A.9})$$

To get (A.9), the following relation should be used:

$$\epsilon_{m_1 \dots m_{D-p}}^{n_1 \dots n_p} \epsilon^{m_1 \dots m_{D-p}}_{k_1 \dots k_p} = \det g \cdot (D-p)! p! \delta_{[k_1}^{[n_1} \dots \delta_{k_p]}^{n_p]} \equiv \det g \cdot (D-p)! p! \delta_{k_1 \dots k_p}^{n_1 \dots n_p}. \quad (\text{A.10})$$

Also, we have

$$dx^{m_1} \wedge \dots \wedge dx^{m_D} = d^D x \epsilon^{m_1 \dots m_D}, \quad (\text{A.11})$$

Taking into account (A.4), (A.8) and (A.10), this implies

$$\frac{1}{2 \cdot p!} \int d^D x \sqrt{-g} (F_{(p)})^2 = (-)^{Dp+p+1} \frac{1}{2} \int_{\mathcal{M}^D} F_{(p)} \wedge *F_{(p)}. \quad (\text{A.12})$$

In our notation

$$\int d^D x \sqrt{-g} \equiv \int_{\mathcal{M}^D} \mathbf{1}, \quad (\text{A.13})$$

and, therefore,

$$\int d^D x \sqrt{-g} R \equiv \int_{\mathcal{M}^D} \mathbf{1} \cdot R. \quad (\text{A.14})$$

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